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The Jacobi group and its holomorphic discrete series representations on Siegel–Jacobi domains

Stefan BERCEANU, Alexandru GHEORGHE

Horia Hulubei National Institute for R&D in Physics and Nuclear Engineering
RO-077125, Bucharest-Magurele P.O.Box MG-6, Romania

Группа Якоби и ее представления голоморфной дискретной серии на областях Зигеля–Якоби

Стефан БЕРЧАНУ, Александру ГЕОРГЕ

Национальный институт исследований и разработок
в области физики и ядерной инженерии им. Хории Хулубея
P.O.Box MG-6, Румыния, Бухарест-Магуреле, RO-077125

Abstract. This is the summary of a part of the talk delivered at the workshop held at the Tambov University in September 2012, reporting several results on Jacobi groups and its holomorphic representations published by the authors.

Keywords: Jacobi group; Siegel–Jacobi domain; canonical automorphy factor; canonical kernel function; scalar holomorphic discrete series

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Аннотация. Эта статья — краткое изложение части лекции, прочитанной на конференции в Тамбовском университете в октябре 2012, излагающее некоторые результаты о группах Якоби и их голоморфных представлениях, полученные авторами.

Ключевые слова: группа Якоби; область Зигеля–Якоби; канонический фактор автоморфности, каноническое ядро (функция); скалярная голоморфная дискретная серия

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Introduction

The Jacobi group is the semidirect product of the real symplectic group with Heisenberg group of adequate dimension [9, 10]. Several generalizations are known [12, 20]. The Jacobi groups are unimodular, nonreductive, algebraic groups of Harish-Chandra type. The Siegel-Jacobi domains are nonreductive symmetric domains associated to the Jacobi groups by the generalized Harish-Chandra embedding [12, 16, 21, 22].

In [1] we have introduced Perelomov coherent states [13] defined on the Siegel-Jacobi disk. Similar constructions have been used previously [11, 14, 17]. The Jacobi group with applications in Quantum Mechanics has been investigated in a series of papers [2–7]. The present note is based manly on [5], where we have not used Perelomov coherent states. The problem of Berezin quantization [8], the fundamental conjecture for homogeneous Kähler manifolds, the classical and quantum evolution on Siegel-Jacobi domains, and the orthonormal base of polynomials in which the Bergman kernel is developed, all summarized in our the talk in accord with [2–7], are not included in this note.

1. Canonical automorphy factor and kernel function

Let $\mathfrak{H}_n$ be the Siegel upper half space of degree n consisting of all symmetric matrices $\Omega \in M_n(\mathbb{C})$ with $\operatorname{Im}\Omega > 0$. The symplectic group $\operatorname{Sp}(n,\mathbb{R})$ of degree n consists of all matrices $\sigma \in M_{2n}(\mathbb{R})$ such that ${}^t\sigma J_n \sigma = J_n$, where

$$\sigma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \; a,b,c,d \in M_n(\mathbb{R}); \quad J_n = \begin{pmatrix} 0 & I_n \\ -I_n & 0 \end{pmatrix}, \tag{1.1}$$

The group $\operatorname{Sp}(n,\mathbb{R})$ acts transitively on $\mathfrak{H}_n$ by $\sigma\Omega = (a\Omega + b)(c\Omega + d)^{-1}$.

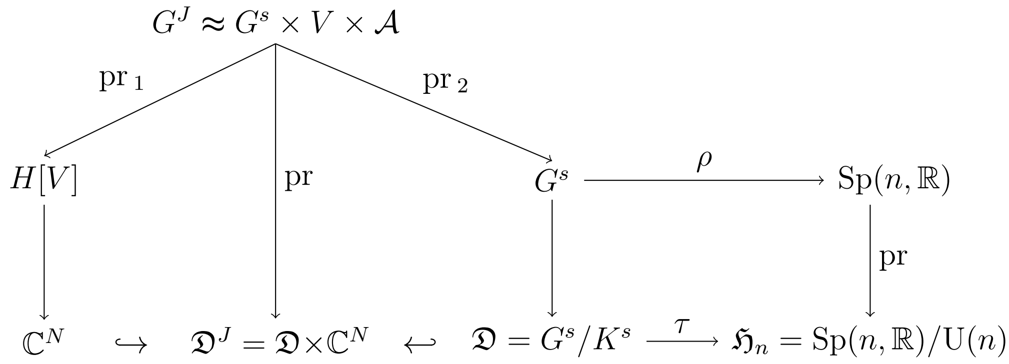
Let G^s be a Zariski connected semisimple real algebraic group of Hermitian type. Let $\mathfrak{D} = G^s/K^s$ be the associated Hermitian symmetric domain, where K^s is a maximal compact subgroup of G^s. Suppose there exist a homomorphism $\rho : G^s \to \operatorname{Sp}(n,\mathbb{R})$ and a holomorphic map $\tau : \mathfrak{D} \to \mathfrak{H}_n$ such that $\tau(gz) = \rho(g)\tau(z)$ for all $g \in G^s$ and $z \in \mathfrak{D}$. The *Jacobi group* G^J [9,12,20] is the semidirect product of G^s and the Heisenberg group $H[V]$ associated with the symplectic $\mathbb{R}$-space V and the nondegenerate alternating bilinear form $D:\ V \times V \to \mathcal{A}$, where $\mathcal{A}$ is the center of $H[V]$. The multiplication operation of $G^J \approx G^s \times V \times \mathcal{A}$ is defined by

$$gg' = (\sigma\sigma', \rho(\sigma)v' + v, \varkappa + \varkappa' + \frac{1}{2}D(v, \rho(\sigma)v')),$$

where $g = (\sigma, v, \varkappa) \in G^J$, $g' = (\sigma', v', \varkappa') \in G^J$. The *Jacobi-Siegel domain* associated to the Jacobi group G^J is defined by $\mathfrak{D}^J = \mathfrak{D} \times \mathbb{C}^N \cong G^J/(K^s \times \mathcal{A})$, where $\dim\ V = 2N$ (cf. [9, 12, 20]). The definitions above are represented in the scheme below.

G^J is an algebraic group of Harish-Chandra type [12, 16, 20].

Following [20] and [12], we obtain [5].



Theorem 1.1. *a) The Jacobi group* G^J *acts transitively on* $\mathfrak{D}^J$ *by*

$$gx = (\sigma w, v_{\sigma w} + {}^t(c\tau(w)+d)^{-1}z),\ \rho(\sigma) = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

where $g = (\sigma, v, \varkappa) \in G^J$ *and* $x = (w,z) \in \mathfrak{D}^J$.

b) The canonical automorphy factor J *for the Jacobi group* G^J *is given by*

$$J(g,x) = (J_1(\sigma,w), 0, J_2(g,x)),$$

where J_1 *is the canonical automorphy factor for* G^s, *and*

$$J_2(g,x) = \varkappa + \frac{1}{2}D(v, v_{\sigma w}) + \frac{1}{2}D(2v+\rho(\sigma)z, J_1(\sigma,w)z).$$

c) The canonical kernel function K *for the Jacobi group* G^J *is given by*

$$K(x,x') = (K_1(w,w'), 0, K_2(x,x')),$$

where K_1 *is the canonical kernel function for* G^s, *and*

$$K_2(x,x') = D(2\overline{z}' + \frac{1}{2}\overline{\tau(w')}z, qz) + \frac{1}{2}D(\overline{z}', q\tau(w)\overline{z}'),\ q = \rho(K_1(w,w'))^{-1}.$$

The Heisenberg group $\mathrm{H}_n(\mathbb{R})$ consists of all elements (λ,μ,κ), where $\lambda,\mu \in M_{1\,n}(\mathbb{R})$, $\kappa \in \mathbb{R}$, with the multiplication law

$$(\lambda,\mu,\kappa)\circ(\lambda',\mu',\kappa') = (\lambda+\lambda', \mu+\mu', \kappa+\kappa'+\lambda\,{}^t\mu' - \mu\,{}^t\lambda').$$

Let $G_n^J = \mathrm{Sp}(n,\mathbb{R}) \ltimes \mathrm{H}_n(\mathbb{R})$ endowed with the following multiplication law:

$$(\sigma,(\lambda,\mu,\kappa))\cdot(\sigma',(\lambda',\mu',\kappa')) = (\sigma\sigma', (\lambda\sigma',\mu\sigma',\kappa)\circ(\lambda',\mu',\kappa')).$$

The *Jacobi group* G_n^J *of degree* n acts transitively on the *Jacobi-Siegel space* $\mathfrak{H}_n^J = \mathfrak{H}_n \times \mathbb{C}^n$ by $g(\Omega,\zeta) = (\Omega_g, \zeta_g)$, where

$$\Omega_g = (a\Omega+b)(c\Omega+d)^{-1},\ \zeta_g = \nu(c\Omega+d)^{-1},\ \nu = \zeta + \lambda\Omega + \mu. \tag{1.2}$$

P r o p o s i t i o n 1.1. *The canonical automorphy factor J_1 and the canonical kernel function K_1 for* $\mathrm{Sp}(n,\mathbb{R})$ *are given by*

$$J_1(\sigma,\Omega) = \begin{pmatrix} {}^t(c\Omega+d)^{-1} & 0 \\ 0 & c\Omega+d \end{pmatrix},$$

$$K_1(\Omega',\Omega) = \begin{pmatrix} 0 & \overline{\Omega}-\Omega' \\ (\Omega'-\overline{\Omega})^{-1} & 0 \end{pmatrix},$$

where $\Omega,\Omega'\in\mathfrak{H}_n$ and $\sigma\in\mathrm{Sp}(n,\mathbb{R})$ is given by (1.1).

The canonical automorphy factor $\theta = J_2(g,(\Omega,\zeta))$ for G^J_n is given by

$$\theta = \kappa + \lambda\,{}^t\zeta + \nu\,{}^t\lambda - \nu(c\Omega+d)^{-1}c\,{}^t\nu, \quad \nu = \zeta+\lambda\Omega+\mu, \tag{1.3}$$

where $g = (\sigma,(\lambda,\,\mu,\kappa))\in G^J_n$, σ is given by (1.1)*, and $(\Omega,\zeta)\in\mathfrak{H}^J_n$.*

The canonical automorphy kernel K_2 for G^J_n is given by

$$K_2\left((\zeta',\Omega'),(\zeta,\Omega)\right) = -\frac{1}{2}(\zeta'-\overline{\zeta})(\Omega'-\overline{\Omega}')^{-1}({}^t\zeta'-{}^t\overline{\zeta}). \tag{1.4}$$

Let $\mathfrak{D}_n$ *be the Siegel disk of degree n* consisting of all symmetric matrices $W\in M_n(\mathbb{C})$ with $I_n - W\overline{W} > 0$. Let $\mathrm{Sp}(n,\mathbb{R})_*$ be the multiplicative group of all matrices $\omega\in M_{2n}(\mathbb{C})$ such that

$$\omega = \begin{pmatrix} p & q \\ \overline{q} & \overline{p} \end{pmatrix}, \quad {}^tp\overline{p} - {}^t\overline{q}q = I_n, \ {}^tp\overline{q} = {}^t\overline{q}p, \quad p,q\in M_n(\mathbb{C}). \tag{1.5}$$

The group $\mathrm{Sp}(n,\mathbb{R})_*$ acts transitively on $\mathfrak{D}_n$ by $\omega W = (pW+q)(\overline{q}W+\overline{p})^{-1}$. Let $K_{n*}\cong \mathrm{U}(n)$ be the maximal compact subgroup of $\mathrm{Sp}(n,\mathbb{R})_*$ consisting of all $\omega\in\mathrm{Sp}(n,\mathbb{R})_*$ given by (1.5) with $p\in\mathrm{U}(n)$ and $q=0$. Then $\mathfrak{D}_n\cong\mathrm{Sp}(n,\mathbb{R})_*/\mathrm{U}(n)$.

Let G^J_{n*} be the Jacobi group consisting of all elements $(\omega,(\alpha,\varkappa))$, where $\omega\in\mathrm{Sp}(n,\mathbb{R})_*$, $\alpha\in\mathbb{C}^n$, $\varkappa\in \mathrm{i}\mathbb{R}$, and endowed with the multiplication law

$$(\omega',(\alpha',\varkappa'))(\omega,(\alpha,\varkappa)) = \left(\omega'\omega, \beta+\alpha, \varkappa+\varkappa'+\beta^t\overline{\alpha}-\overline{\beta}^t\alpha\right),$$

where $(\omega,(\alpha,\varkappa)),(\omega',(\alpha',\varkappa'))\in G^J_{n*}$ $\beta = \alpha'p+\overline{\alpha}'\overline{q}$, and ω is given by (1.5).

The Heisenberg group $\mathrm{H}_n(\mathbb{R})_*$ consists of all elements $(I_n,(\alpha,\varkappa))\in G^J_{n*}$, with $\alpha\in\mathbb{C}^n$, $\varkappa\in\mathrm{i}\mathbb{R}$. The center $\mathcal{A}_*\cong\mathbb{R}$ of $\mathrm{H}_n(\mathbb{R})_*$ consists of all elements $(I_n,(0,\varkappa))\in G^J_{n*}$, $\varkappa\in\mathrm{i}\mathbb{R}$. There exists an isomorphism $\Theta: G^J_n\to G^J_{n*}$ given by $\Theta(g) = g_*$, $g=(\sigma,(\lambda,\mu,\kappa))\in G^J_n$, $g_* = (\omega,(\alpha,\varkappa))\in G^J_{n*}$,

$$\sigma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \ \omega = \begin{pmatrix} p_+ & p_- \\ \overline{p}_- & \overline{p}_+ \end{pmatrix},$$

$$p_\pm = \frac{1}{2}(a\pm d) \pm \frac{\mathrm{i}}{2}(b\mp c), \quad \alpha = \frac{1}{2}(\lambda+\mathrm{i}\mu), \quad \varkappa = -\mathrm{i}\frac{\kappa}{2}.$$

Let $\mathfrak{D}^J_n = \mathfrak{D}_n\times\mathbb{C}^n\cong G^J_{n*}/(\mathrm{U}(n)\times\mathbb{R})$ be *the Siegel-Jacobi disk of degree n.* G^J_{n*} acts transitively on $\mathfrak{D}^J_n$ by $g_*(W,z) = (W_{g_*},z_{g_*})$, where

$$W_{g_*} = (pW+q)(\overline{q}W+\overline{p})^{-1}, \ z_{g_*} = (z+\alpha W+\overline{\alpha})(\overline{q}W+\overline{p})^{-1}. \tag{1.6}$$

We now consider a partial Cayley transform of the Siegel-Jacobi disk $\mathfrak{D}_n^J$ onto the Siegel-Jacobi space $\mathfrak{H}_n^J$ which gives a partially bounded realization of $\mathfrak{H}_n^J$ [22]. The *partial Cayley transform* $\phi : \mathfrak{D}_n^J \to \mathfrak{H}_n^J$ is defined by

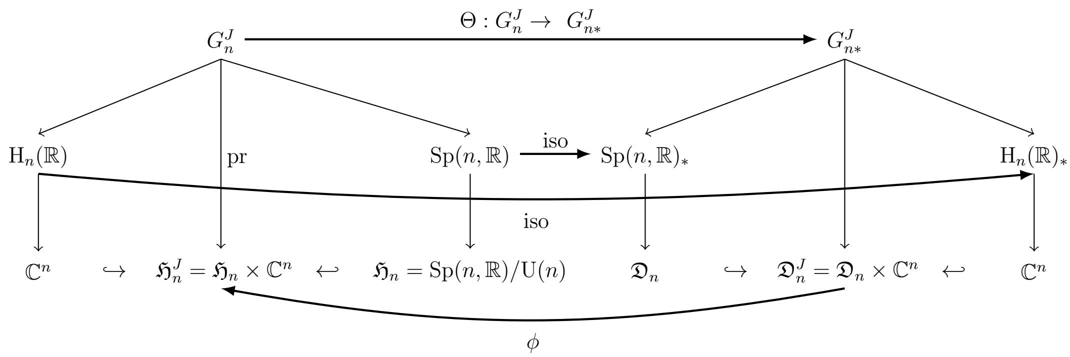
$$\Omega = \mathrm{i}(I_n + W)(I_n - W)^{-1}, \quad \zeta = 2\,\mathrm{i}\, z(I_n - W)^{-1}, \tag{1.7}$$

where $(\zeta, \Omega) = \phi((W, z))$ and $(W, z) \in \mathfrak{D}_n^J$. Thr map ϕ is a biholomorphic map which satisfies $g\phi = \phi g_*$ for any $g \in G_n^J$ and $g_* = \Theta(g)$ [22].

The inverse partial Cayley transform $\phi^{-1} : \mathfrak{H}_n^J \to \mathfrak{D}_n^J$ is given by

$$W = (\Omega - \mathrm{i}I_n)(\Omega + \mathrm{i}I_n)^{-1}, \quad z = \zeta(\Omega + \mathrm{i}I_n)^{-1}. \tag{1.8}$$

The situation is summarized in the diagram below.



P r o p o s i t i o n 1.2. *The canonical automorphy factor* J_{1*} *and the canonical kernel function* K_{1*} *for* $\mathrm{Sp}(n,\mathbb{R})_*$ *are given by*

$$J_{1*}(\omega, W) = \begin{pmatrix} {}^t(\overline{q}W + \overline{p})^{-1} & 0 \\ 0 & \overline{q}W + \overline{p} \end{pmatrix},$$

$$K_{1*}(W', W) = \begin{pmatrix} I_n - W'\overline{W} & 0 \\ 0 & {}^t(I_n - W'\overline{W})^{-1} \end{pmatrix},$$

where $W, W' \in \mathfrak{D}_n$ *and* $\omega \in \mathrm{Sp}(n,\mathbb{R})_*$ *is given by* (1.5).

The canonical automorphy factor $\theta_* = J_2(g_{n*}, (W, z))$ *for* G^J_{n*} *is given by*

$$\theta_* = \kappa_* + z\,{}^t\alpha + \nu_*\,{}^t\alpha - \nu_*(\overline{q}W + \overline{p})^{-1}\overline{q}\,{}^t\nu_*, \quad \nu_* = z + \alpha W + \overline{\alpha}, \tag{1.9}$$

where $g_* = (\omega, (\alpha, \varkappa)) \in G^J_{n*}$, ω *is given by* (1.5), *and* $(W, z) \in \mathfrak{D}_n^J$.

The canonical automorphy kernel for G^J_{n*} *is given by*

$$K_{2*}((W', z'), (W, z)) = A(W', z'; W, z),$$

where $(W, z), (W', z') \in \mathfrak{D}_n^J$, *and*

$$A(W', z'; W, z) = (\overline{z} + \frac{1}{2}z'\overline{W})(I_n - W'\overline{W})^{-1}\,{}^tz' + \frac{1}{2}\overline{z}(I_n - W'\overline{W})^{-1}W'\,{}^t\overline{z}. \tag{1.10}$$

2. Scalar holomorphic discrete series

Consider the Jacobi group G^J_n. Let δ be a rational representation of $GL(n,\mathbb{C})$ such that $\delta|_{\mathrm{U}(n)}$ is a scalar irreducible representation of the unitary group $\mathrm{U}(n)$ with highest weight k, $k\in\mathbb{Z}$, and $\delta(A)=(\det A)^k$ [23]. Let $m\in\mathbb{R}$. Let $\chi=\delta\otimes\overline{\chi}^m$, where the central character χ^m of $\mathcal{A}\cong\mathbb{R}$ is defined by $\chi^m(\kappa)=\exp(2\pi i m\kappa)$, $\kappa\in\mathcal{A}$. Any scalar holomorphic irreducible representation of G^J_n is characterized by an index m and a weight k. Suppose $m>0$ and $k>n+1/2$.

Let $\mathcal{H}^{mk}$ denote the Hilbert space of all holomorphic functions $\varphi\in\mathcal{O}(\mathfrak{H}^J_n)$ such that $\|\varphi\|_{\mathfrak{H}^J_n}<\infty$ with the inner product defined by [18]

$$(\varphi,\psi)_{\mathfrak{H}^J_n}=C\int_{\mathfrak{H}^J_n}\varphi(\Omega,\zeta)\overline{\psi(\Omega,\zeta)}\,\mathcal{K}^{mk}(\Omega,\zeta)^{-1}d\mu(\Omega,\zeta),$$

where C is a positive constant, $(\Omega,\zeta)\in\mathfrak{H}^J_n$ and the G^J_n-invariant measure on $\mathfrak{H}^J_n$ is given by

$$d\mu(\Omega,\zeta)=(\det Y)^{-n-2}\prod_{1\leqslant i\leqslant n}d\xi_i\,d\eta_i\prod_{1\leqslant j\leqslant k\leqslant n}dX_{jk}\,dY_{jk}.$$

Here $\xi=\mathrm{Re}\,\zeta$, $\eta=\mathrm{Im}\,\zeta$, $X=\mathrm{Re}\,\Omega$, $Y=\mathrm{Im}\,\Omega$.

The kernel function $\mathcal{K}^{mk}$ is defined by [18]

$$\mathcal{K}^{mk}(\Omega,\zeta)=\mathsf{K}^{mk}((\Omega,\zeta),(\Omega,\zeta))=\exp\left(4\pi m\eta Y^{-1}\,{}^t\eta\right)(\det Y)^k,$$

$$\mathsf{K}^{mk}((\zeta',\Omega'),(\zeta,\Omega))=\left(\det(\frac{i}{2}\overline{\Omega}-\frac{i}{2}\Omega')\right)^{-k}\exp(2\pi i m K\left((\zeta',\Omega'),(\zeta,\Omega)\right)),$$

where K is given by (1.4).

Let π^{mk} be the unitary representation of G^J_n on $\mathcal{H}^{mk}$ defined by [18]

$$\left(\pi^{mk}(g^{-1})\varphi\right)(\Omega,\zeta)=\mathcal{J}^{mk}(g,(\Omega,\zeta))\varphi(\Omega_g,\zeta_g),$$

where $\varphi\in\mathcal{H}^{mk}$, $g\in G^J_n$, $(\Omega,\zeta)\in\mathfrak{H}^J_n$ and $(\Omega_g,\zeta_g)\in\mathfrak{H}^J_n$ is given by (1.2).

The automorphic factor $\mathcal{J}^{mk}$ for G^J_n is defined by [18]

$$\mathcal{J}^{mk}(g,(\zeta,\Omega))=(\det(c\Omega+d))^{-k}\exp(2\pi i m\theta),$$

where θ is given by (1.3) and σ is given by (1.1).

Takase proved the following theorem [18, 19]:

Theorem 2.1. *Suppose $k>n+1/2$. Then $\mathcal{H}^{mk}\neq\{0\}$ and π^{mk} is an irreducible unitary representation of G^J_n which is square integrable modulo center.*

Let $\mathcal{H}^{mk}_*$ denote the complex pre-Hilbert space of all $\psi\in\mathcal{O}(\mathfrak{D}^J_n)$ such that $\|\psi\|_{\mathfrak{D}^J_n}<\infty$ with the inner product defined by

$$(\psi_1,\psi_2)_{\mathfrak{D}^J_n}=C_*\int_{\mathfrak{D}^J_n}\psi_1(W,z)\,\overline{\psi_2(W,z)}\,\left(\mathcal{K}^{mk}_*(W,z)\right)^{-1}d\,\nu(W,z),$$

where C_* is a positive constant, $(z,W)\in\mathfrak{D}^J_n$,

$$\mathcal{K}^{mk}_*(W,z)=\left(\det(I_n-W\overline{W})\right)^{-k}\exp(8\pi mA(W,z)),$$

and $A(W,z)=K_{2*}((W,z),(W,z))$ can be written as

$$A(W,z)=(\overline{z}+\frac{1}{2}z\overline{W})(I_n-W\overline{W})^{-1}\,{}^tz+\frac{1}{2}\overline{z}(I_n-W\overline{W})^{-1}W\,{}^t\overline{z}.$$

and the G^J_n-invariant measure on $\mathfrak{D}^J_n$ is [22]

$$d\nu(W,z)=(\det(1-W\overline{W}))^{-n-2}\prod_{i=1}^{n}d\,\mathrm{Re}z_i\,d\,\mathrm{Im}z_i\prod_{1\leq j\leqslant k\leqslant n}d\,\mathrm{Re}\,W_{jk}d\,\mathrm{Im}W_{jk.}$$

According with [15, 22], and (1.10), the kernel function $\mathcal{K}^{mk}_*$ is given by

$$\mathcal{K}^{mk}_*(W,z)=\mathsf{K}^{mk}_*((W,z),(W,z)),$$

where

$$\mathsf{K}^{mk}_*((z,W),(z',W'))=\left(\det(I_n-W'\overline{W})\right)^{-k}\exp\left(8\pi mA(W',z';W,z)\right).$$

We now introduce the map $g_*\longmapsto\pi^{mk}_*(g_*)$, where $\pi^{mk}_*(g_*):\ \mathcal{H}^{mk}_*\to\ \mathcal{H}^{mk}_*$ is defined by

$$\left(\pi^{mk}_*(g_*^{-1})\psi\right)(z,W)=J^{mk}_*(g_*,(z,W))\psi(z_{g_*},W_{g_*}),$$

$\psi\in\mathcal{H}^{mk}_*$, $g_*=(\omega,(\alpha,\varkappa))\in G^J_{n*}$, $(z,W)\in\mathfrak{D}^J_n$, and $(z_{g_*},W_{g_*})\in\mathfrak{D}^J_n$ is given by (1.6). The automorphic factor J^{mk}_* for G^J_{n*} is defined by [15, 22]

$$J^{mk}_*(g_*,(z,W))=\exp(2\pi\mathrm{i}m\theta_*)\left(\det(\overline{q}W+\overline{p})\right)^{-k},$$

where θ_* is given by (1.9) and ω given by (1.5).

P r o p o s i t i o n 2.1. *Suppose* $m>0$, $k>n+1/2$, *and* $C=2^{n(n+3)}C_*$. *Then*

a) $\mathcal{H}^{mk}_*\neq\{0\}$ *and* π^{mk}_* *is an irreducible unitary representation of* G^J_{n*} *on the Hilbert space* $\mathcal{H}^{mk}_*$ *which is square integrable modulo center.*

b) There exists the unitary isomorphism $T^{mk}_*:\mathcal{H}^{mk}_*\to\mathcal{H}^{mk}$ *given by*

$$\varphi(\Omega,\zeta)=\psi\ (W,z)\left(\det(I_n-W)\right)^k\exp(4\pi mz(I_n-W)^{-1}\,{}^tz),$$

where $\psi\in\mathcal{H}^{mk}_*$, $\varphi=T^{mk}_*(\psi)$, $(W,z)\in\mathfrak{D}^J_n$, $(\Omega,\zeta)=\phi((-W,z))\in\mathfrak{H}^J_n$, *and* ϕ *is given by* (1.7).

The inverse isomorphism $T^{mk}:\mathcal{H}^{mk}\to\mathcal{H}^{mk}_*$ *is given by*

$$\psi\ (W,z)=\varphi(\Omega,\zeta)\ \left(\det(I_n-\mathrm{i}\Omega)\right)^k\exp\left(2\pi m\zeta(I_n-\mathrm{i}\Omega)^{-1}\,{}^t\zeta\right),$$

where $\psi\in\mathcal{H}^{mk}_*$, $\psi=T^{mk}(\varphi)$, $(\Omega,\zeta)\in\mathfrak{H}^J_n$, $(-W,z)=\phi^{-1}\left((\Omega,\zeta)\right)\in\mathfrak{D}^J_n$, *and* ϕ^{-1} *is given by* (1.8).

c) The representations π^{mk} *and* π^{mk}_* *are unitarily equivalent.*

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References

[1] S. Berceanu, "A holomorphic representation of the Jacobi algebra", *Reviews in Mathematical Physics*, **18**:2 (2006), 163–199, arXiv: math/0408219v3.

[2] S. Berceanu, "Coherent states associated to the Jacobi group – a variation on a theme by Erich Kähler", *Journal of Geometry and Symmetry in Physics*, **9** (2007), 1–8 crossref.

[3] S. Berceanu, "A holomorphic representation of Jacobi algebra in several dimensions", *The Theta Foundation*, Perspectives in Operator Algebras and Mathematical Physics (Bucharest, August 10–17, 2005), Theta Series in Advanced Mathematics, **11**, eds. F. P. Boca, R. Purice, S. Strătilă, Conference Proceedings, 2008, 1–25.

[4] S. Berceanu, A. Gheorghe, "Applications of the Jacobi group to Quantum Mechanics", *Romanian Journal of Physics*, **53**:9–10 (2008), 1013–1021.

[5] S. Berceanu and A. Gheorghe, "On the geometry of Siegel-Jacobi domains", *International Journal of Geometric Methods in Modern Physics*, **8** (2011), 1783–1798.

[6] S. Berceanu, "A convenient coordinatization of Siegel–Jacobi domains", *Reviews in Mathematical Physics*, **24**:10 (2012), 1–38 crossref.

[7] S. Berceanu, "Consequences of the fundamental conjecture for the motion on the Siegel-Jacobi disk", *International Journal of Geometric Methods in Modern Physics*, **10**:01 (2013), 1–18 crossref.

[8] F. A. Berezin, "Quantization in complex symmetric spaces", *Math. USSR-Izv.*, **9**:2 (1975), 341–379 crossref.

[9] R. Berndt and R. Schmidt, *Progress in Mathematics*: *Elements of the Representation Theory of the Jacobi Group*, **163**, Birkhäuser, Basel, 1998, 216 pp.

[10] M. Eichler and D. Zagier, *Progress in Mathematics*: *The Theory of Jacobi Forms*, **55**, Birkhäuser, Boston, 1985.

[11] P. Kramer and M. Saraceno, "Semicoherent states and the group $\mathrm{ISp}(2,\mathbb{R})$", *Physica A: Statistical Mechanics and its Applications*, **114**:1–3 (1982), 448–453 crossref.

[12] M. H. Lee, "Theta functions on hermitian symmetric domains and Fock representations", *Journal of the Australian Mathematical Society*, **74**:2 (2003), 201–234 crossref.

[13] A. M. Perelomov, *Generalized Coherent States and their Applications*, Springer, Berlin, 1986.

[14] C. Quesne, "Vector coherent state theory of the semidirect sum Lie algebras wsp $(2N,\mathbb{R})$", *Journal of Physics A: Mathematical and General*, **23**:6 (1990), 847–862.

[15] I. Satake, "Unitary representations of a semi-direct products of Lie groups on $\overline{\partial}$-cohomology spaces", *Mathematische Annalen*, **190**:3 (1971), 177–202.

[16] I. Satake, *Algebraic Structures of Symmetric Domains*, Princeton University Press, Princeton, 1980.

[17] K. Shuman, "Complete signal processing bases and the Jacobi group", *Journal of Mathematical Analysis and Applications*, **278**:1 (2003), 203–213 crossref.

[18] K. Takase, "A note on automorphic forms", *J. Reine Angew. Math.*, **409** (1990), 138–171.

[19] K. Takase, "On unitary representations of Jacobi groups", *J. Reine Angew. Math.*, **430** (1992), 130–149.

[20] K. Takase, "On Siegel modular forms of half-integral weights and Jacobi forms", *Trans. Amer. Math. Soc.*, **351**:2 (1999), 735–780.

[21] J.-H. Yang, "The method of orbits for real Lie groups", *Kyungpook Mathematical Journal*, **42**:2 (2002), 199–272.

[22] J.-H. Yang, "A partial Cayley transform for Siegel-Jacobi disk", *Journal of the Korean Mathematical Society*, **45** (2008), 781–794.

[23] C. Ziegler, "Jacobi forms of higher degree", *Abhandlungen aus dem Mathematischen Seminar der Universitat Hamburg*, **59**:1 (1989), 191–224.

Information about the authors

Stefan Berceanu, Professor. Horia Hulubei National Institute for R&D in Physics and Nuclear Engineering, Bucharest-Magurele, Romania. E-mail: Berceanu@theory.nipne.ro

Alexandru Gheorghe, Professor. Horia Hulubei National Institute for R&D in Physics and Nuclear Engineering, Bucharest-Magurele, Romania.

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Corresponding author:
Stefan Berceanu
E-mail: Berceanu@theory.nipne.ro

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Информация об авторах

Берчану Стефан, профессор. Национальный институт исследований и разработок в области физики и ядерной инженерии им. Хории Хулубея, г. Бухарест-Магуреле, Румыния. E-mail: Berceanu@theory.nipne.ro

Георге Александру, профессор. Национальный институт исследований и разработок в области физики и ядерной инженерии им. Хории Хулубея, г. Бухарест-Магуреле, Румыния.

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Для контактов:
Берчану Стефан
E-mail: Berceanu@theory.nipne.ro

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